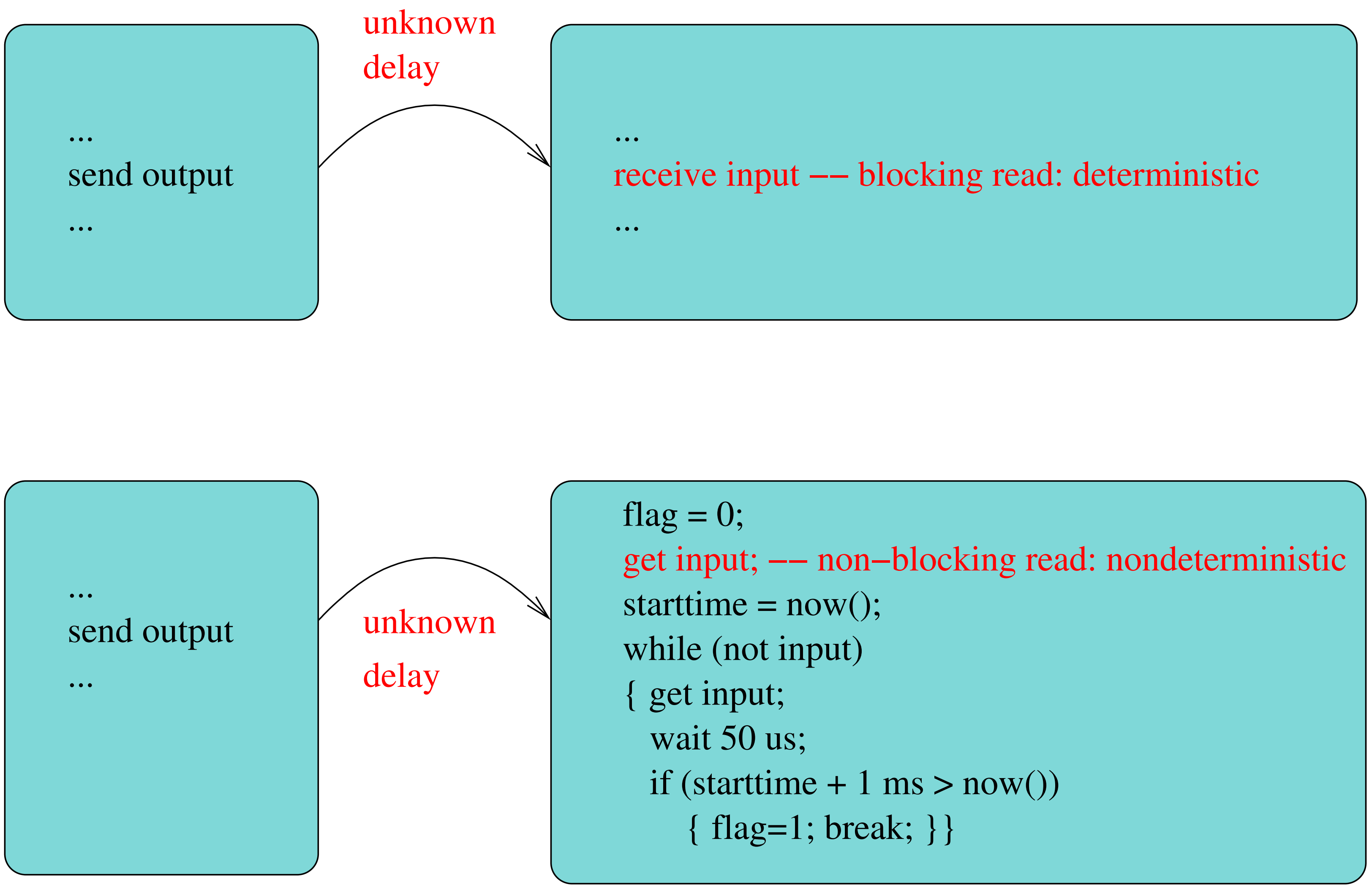
System Modeling

- Introduction
- Rugby Meta-Model
- Finite State Machines
- Petri Nets
- Untimed Model of Computation
- Synchronous Model of Computation
- Timed Model of Computation
- Integration of Computational Models
- Tightly Coupled Process Networks
- **Nondeterminism and Probability**

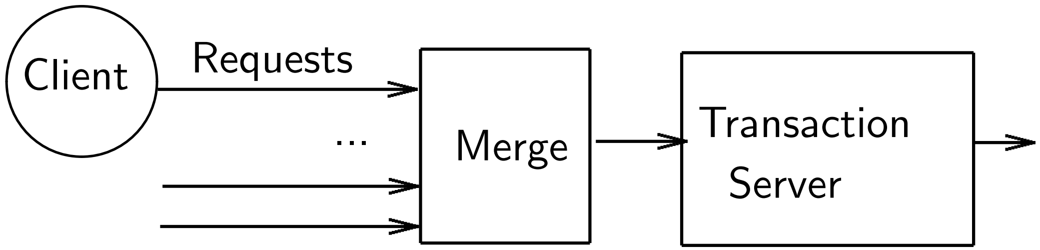
Purpose of Nondeterminism

- Descriptive Purpose
  - ⋆ Modeling of an entity with limited knowledge (e.g. unknown communication time)
  - ⋆ Nondeterminism as an abstraction mechnism;
  - ⋆ All possibilities must be exercised for validation;
- Constraining Purpose
  - ⋆ The specification gives freedom to the implementation
  - ⋆ One possibility must be implemented

Modeling Unknown Communication Delay



Unspecified Merge of Inputs



The Merge model should have the following properties:

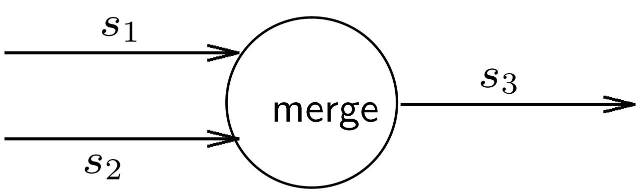
- Requests are not ordered by the Merge
- Absent request should not block the Merge
- The Merge should be fair

Merge as an Untimed Process Relation

$$\text{merge}(s_1, \langle\rangle) = s_1$$

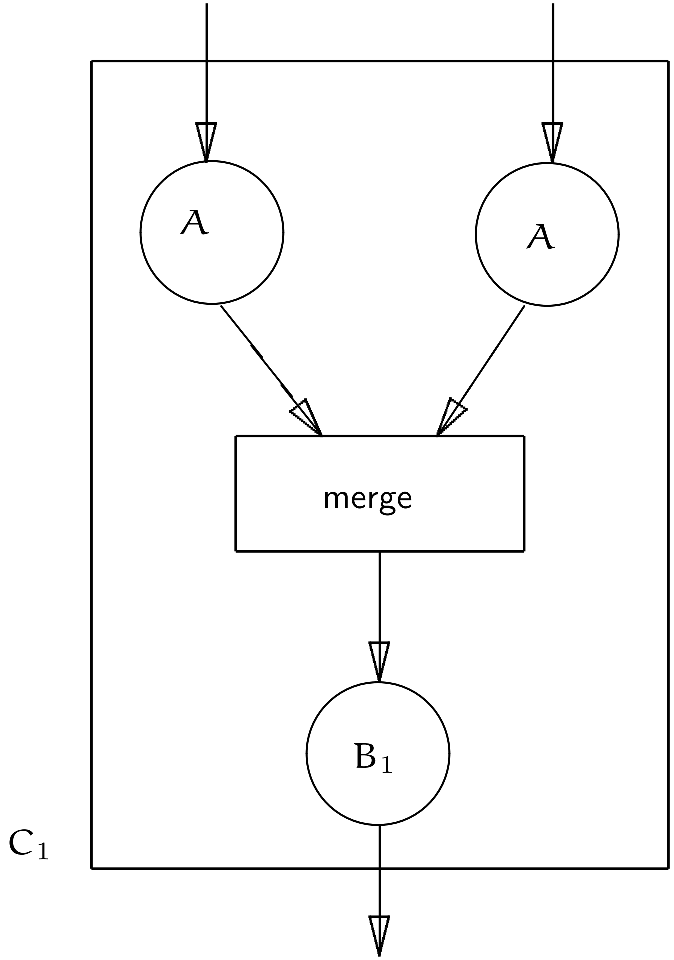
$$\text{merge}(\langle\rangle, s_2) = s_2$$

$$\text{merge}(e_1 \oplus s_1, e_2 \oplus s_2) = \{e_1 \oplus s_3 : s_3 \in \text{merge}(s_1, e_2 \oplus s_2)\}$$
$$\cup \{e_2 \oplus s_3 : s_3 \in \text{merge}(e_1 \oplus s_2, s_2)\}$$



s_1	s_2	s_3
$\langle a, b\rangle$	$\langle 1, 2\rangle$	$\{\langle a,b,1,2\rangle, \langle a,1,b,2\rangle, \langle a,1,2,b\rangle,$ $\langle 1,a,b,2\rangle, \langle 1,a,2,b\rangle, \langle 1,2,a,b\rangle\}$
$\langle c, d\rangle$	$\langle 3, 4\rangle$	$\{\langle c,d,3,4\rangle, \langle c,3,d,4\rangle, \langle c,3,4,d\rangle,$ $\langle 3,c,d,4\rangle, \langle 3,c,4,d\rangle, \langle 3,4,c,d\rangle\}$

The Brock - Ackerman Anomaly - 1



$$\mathrm{C}_1(s_1, s_2) = \mathrm{B}_1(\mathsf{merge}(\mathcal{A}(s_1), \mathcal{A}(s_2)))$$

$$\mathcal{A}(\langle\rangle) = \langle\rangle$$

$$\mathcal{A}(e \oplus s) = \langle e, e\rangle$$

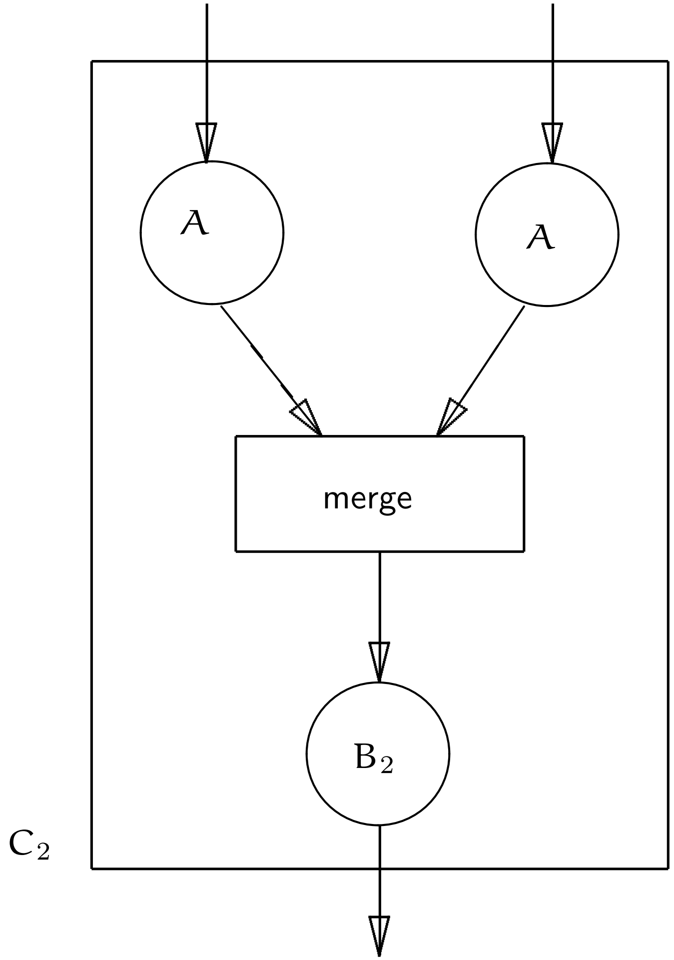
$$\mathrm{B}_1(\langle\rangle) = \langle\rangle$$

$$\mathrm{B}_1(\langle e\rangle \oplus s) = \langle e\rangle \oplus f_1(s)$$

$$f_1(\langle\rangle) = \langle\rangle$$

$$f_1(\langle e'\rangle \oplus s) = \langle e'\rangle$$

The Brock - Ackerman Anomaly - 2



$$\mathrm{C}_2(s_1, s_2) = \mathrm{B}_2(\mathsf{merge}(\mathrm{A}(s_1), \mathrm{A}(s_2)))$$

$$\mathrm{A}(\langle\rangle) = \langle\rangle$$

$$\mathrm{A}(e \oplus s) = \langle e, e\rangle$$

$$\mathrm{B}_2(\langle\rangle) = \langle\rangle$$

$$\mathrm{B}_2(\langle e\rangle \oplus s) = f_2(e, s)$$

$$f_2(e, \langle\rangle) = \langle\rangle$$

$$f_2(e, \langle e'\rangle \oplus s) = \langle e, e'\rangle$$

The Brock - Ackerman Anomaly - 3

$C_1(s_1, s_2) = B_1(\text{merge}(A(s_1), A(s_2)))$

$A(\langle\rangle) = \langle\rangle$

$A(e \oplus s) = \langle e, e\rangle$

$B_1(\langle\rangle) = \langle\rangle$

$B_1(\langle e\rangle \oplus s) = \langle e\rangle \oplus f_1(s)$

$f_1(\langle\rangle) = \langle\rangle$

$f_1(\langle e'\rangle \oplus s) = \langle e'\rangle$

$C_2(s_1, s_2) = B_2(\text{merge}(A(s_1), A(s_2)))$

$A(\langle\rangle) = \langle\rangle$

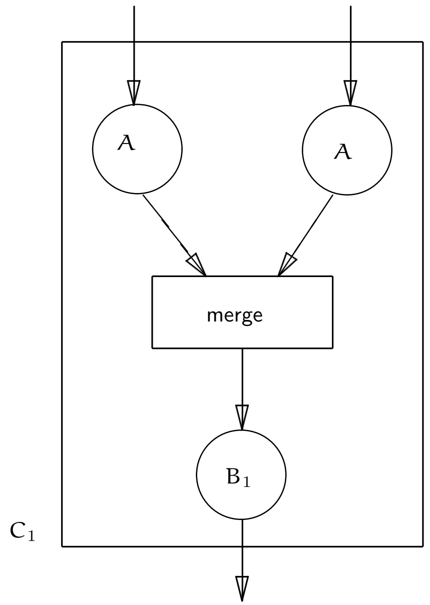
$A(e \oplus s) = \langle e, e\rangle$

$B_2(\langle\rangle) = \langle\rangle$

$B_2(\langle e\rangle \oplus s) = f_2(e, s)$

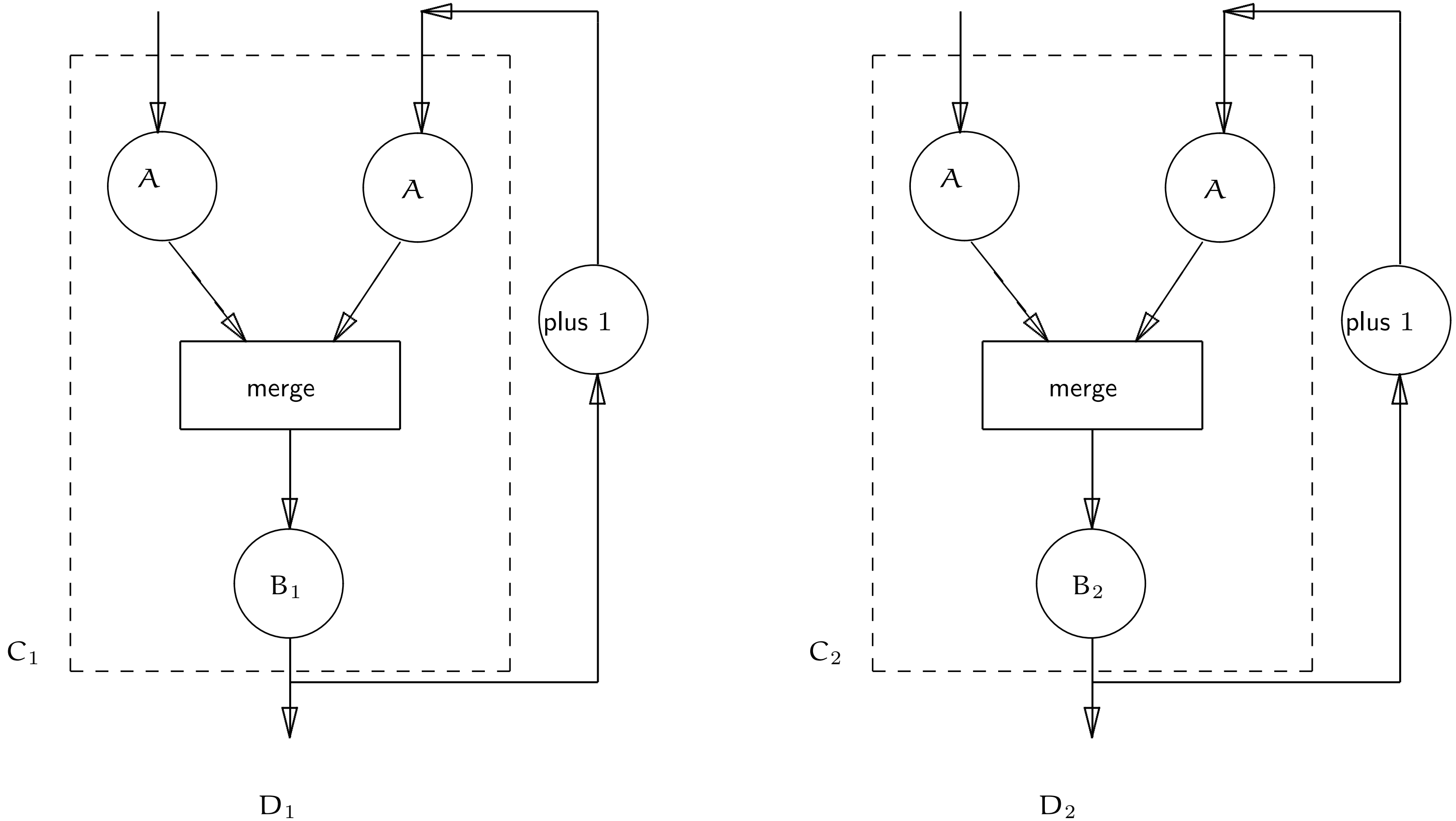
$f_2(e, \langle\rangle) = \langle\rangle$

$f_2(e, \langle e'\rangle \oplus s) = \langle e, e'\rangle$



$$C_i(\langle\rangle, \langle\rangle) = \langle\rangle$$
$$C_i(e \oplus s, \langle\rangle) = \langle e, e\rangle$$
$$C_i(\langle\rangle, e' \oplus s') = \langle e', e'\rangle$$
$$C_i(e \oplus s, e' \oplus s') = \{\langle e, e\rangle, \langle e, e'\rangle, \langle e', e\rangle, \langle e', e'\rangle\}$$

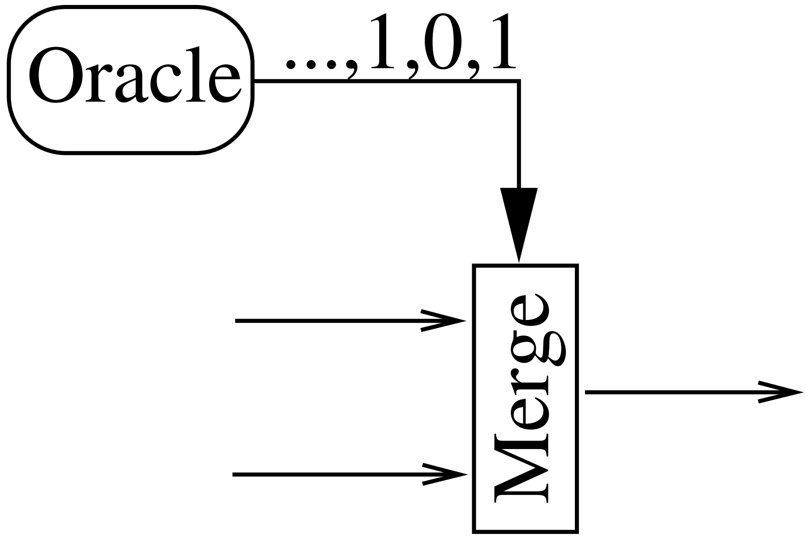
The Brock - Ackerman Anomaly - 4



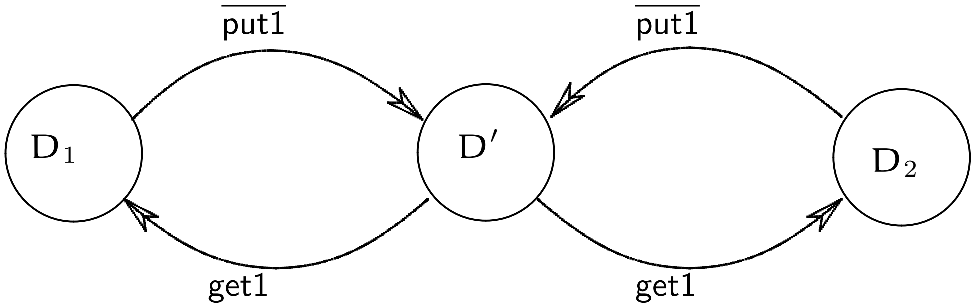
$D_1(\langle 5 \rangle) = \{\langle 5, 5 \rangle, \langle 5, 6 \rangle\}$

$D_2(\langle 5 \rangle) = \{\langle 5, 5 \rangle\}$

Oracle Based Nondeterminism



Weak Determinacy



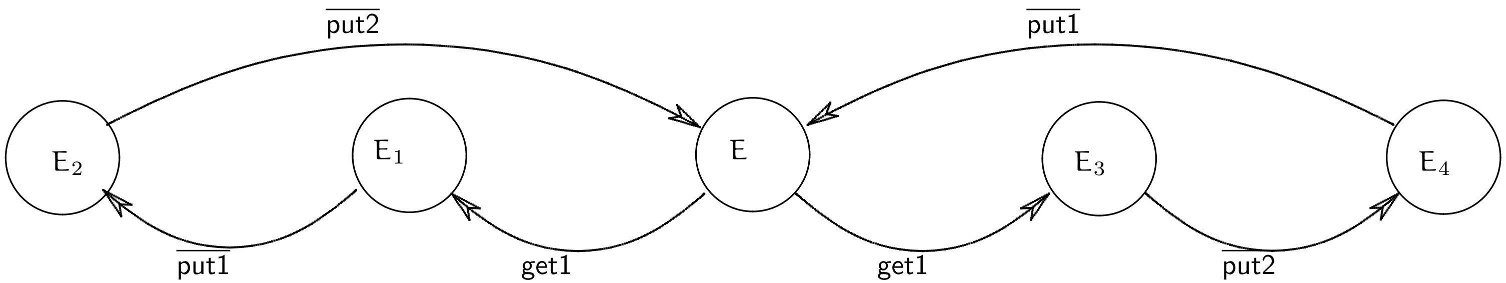
$$\mathrm{D}' \stackrel{\mathrm{def}}{=} \mathsf{get1}.\mathrm{D}_1 + \mathsf{get1}.\mathrm{D}_2$$

$$\mathrm{D}_1 \stackrel{\mathrm{def}}{=} \overline{\mathsf{put1}}.\mathrm{D}'$$

$$\mathrm{D}_2 \stackrel{\mathrm{def}}{=} \overline{\mathsf{put1}}.\mathrm{D}'$$

If a system behaves deterministically as far as it can be observed from the environment, the system is weakly determinate.

Weak Confluence



If, for every two possible actions, the occurrence of one can never preclude the other, the system is weakly confluent.

Modeling with Non-determinisitic Components

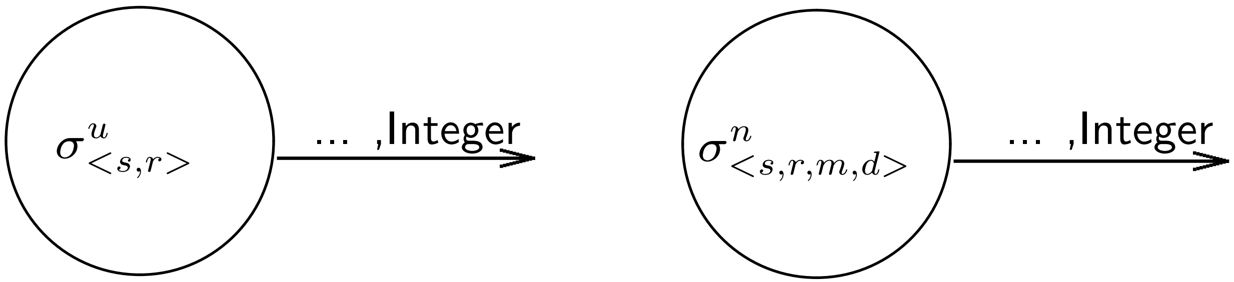
- Weak determinacy or weak confluence is desirable
- Design rules to achieve determinacy and and confluence are necessary
- Example rules:
  - ⋆ Deploying blocking read ensures deterministic system behaviour in the presence of non-deterministic communication and computation time.
  - ⋆ Process composition of processes with disjoint inputs preserves weak confluence.

Modeling Approach with Deterministic Components

- Restrict all modeling elements to be determinsitic
- Constrain implementation such that it is equivalent with the specification model
- Examples:
  - ⋆ Blocking read
  - ⋆ Synchronous assumptions

The σ Process

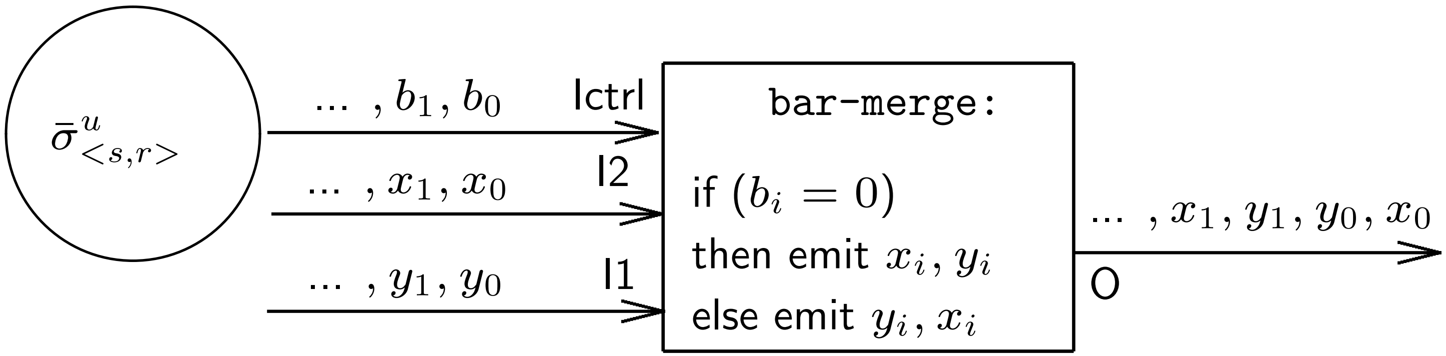
Stochastic processes can be used for descriptive and constraining purpose.



$$\begin{aligned}
\texttt{sigmauC}(min, max, w_0) \quad &= \quad \sigma^u_{<w_0,[min,max]>} \\
\text{where} \quad & \quad \sigma^u_{<w_0,[min,max]>}() = s' \\
& \quad \forall e' \in s' : min \leq \mathsf{val}(e') \leq max \\
& \quad \wedge \forall v \in [min, max] : p_v(e') = \frac{1}{max - min + 1} \\
& \quad p_v(e') \text{...probability that the value of } e' \text{ is } v
\end{aligned}$$

Usage of Stochastic Processes

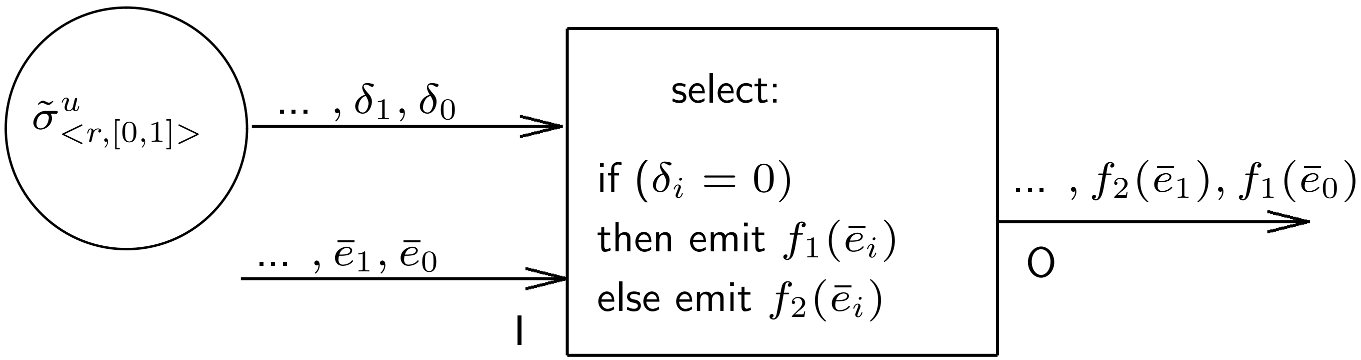
- An implemented sigma bar process $\bar{\sigma}$ can generate any of the possible outputs of a σ.
- An implemented sigma tilde processes $\tilde{\sigma}$ must exhibit the statistical properties of $\tilde{\sigma}$.
- Verification follows the interpretation of implementation for σ processes.



Select Based Process Constructors

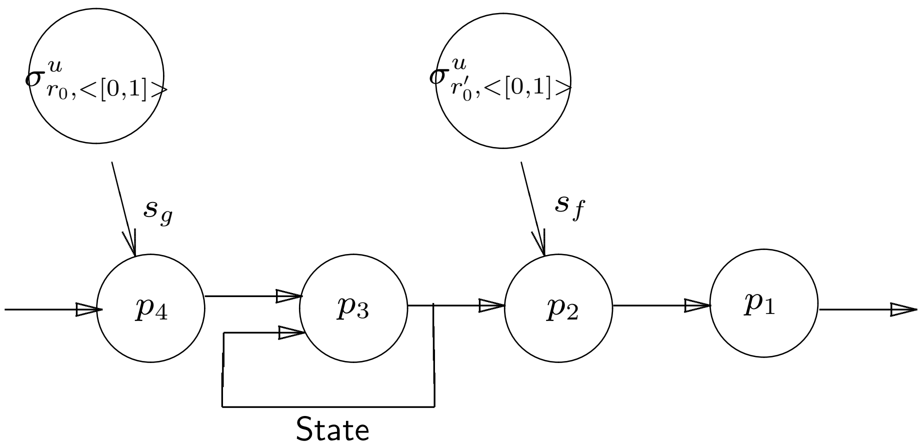
$$
\begin{aligned}
\text{select}(\delta, f, g) &= f \quad \text{if} \quad \delta = 0 \\
&= g \quad \text{if} \quad \delta = 1
\end{aligned}
$$

$$
\begin{aligned}
\texttt{selMapS}(f_1, f_2, r_0) &= p \\
\text{with} \quad & p(\bar{s}) = p_1(p_2(\sigma^u_{<r_0,[0,1]>}, \bar{s})) \\
& p_1 = \texttt{mapS}(f) \\
& p_2 = \texttt{zipS}() \\
& f((\delta, \bar{e})) = \text{select}(\delta, f_1, f_2)(\bar{e})
\end{aligned}
$$



Select Based Moore Constructor

$$
\begin{aligned}
\texttt{selMooreS}(g_1, g_2, f_1, f_2, r_0, r'_0, w_0) \quad &= \quad p \\
\text{with} \quad &= \quad p(\bar{s}) = p_1(p_2(s_f, p_3(p_4(s_g, \bar{s})))) \\
& \quad\quad s_f = \sigma^u_{<r'_0,[0,1]>} \\
& \quad\quad s_g = \sigma^u_{<r_0,[0,1]>} \\
& \quad\quad p_1 = \texttt{mapS}(f) \\
& \quad\quad p_2 = \texttt{zipS}() \\
& \quad\quad p_3 = \texttt{scanS}(g, w_0) \\
& \quad\quad p_4 = \texttt{zipS}() \\
& \quad\quad f((\delta, \bar{e})) = \mathsf{select}(\delta, f_1, f_2)(\bar{e}) \\
& \quad\quad g(w, (\delta, \bar{e})) = \mathsf{select}(\delta, g_1, g_2)(w, \bar{e})
\end{aligned}
$$



Consolidation Based Constructor

$$
\begin{aligned}
\texttt{conMooreS}(g, f, r_0, r'_0, w_0) &= p \\
\text{with} \quad & p(\bar{s}) = p_1(p_2(s_f, p_3(p_4(s_g, \bar{s})))) \\
& s_f = \sigma^u_{<r'_0,[0,1]>} \\
& s_g = \sigma^u_{<r_0,[0,1]>} \\
& p_1 = \texttt{mapS}(f) \\
& p_2 = \texttt{zipS}() \\
& p_3 = \texttt{scanS}(g, w_0) \\
& p_4 = \texttt{zipS}()
\end{aligned}
$$

Sumary

- Descriptive Purpose - Constraining Purpose
- Modeling the unknwon
- Deterministic and Nondeterministic Merge
- Modeling Nondeterminism with History Relations
- Nondeterministic Components and Deterministic Systems
- Usage of stochastic processes
  - ⋆ To model the system's environment;
  - ⋆ To constrain the system under design;
  - ⋆ For modeling testbenches and input stimuli

System Modeling - Sumary

- Modeling Basics: State, Event, Next State and Output Encoding Function, Continuous and Discrete State/Time Models
- Rugby Meta-model
- Finite State Machines
- Petri Nets
- Untimed Model of Computation
- Synchronous Model of Computation
- Discrete Time Model of Computation
- Hierarchical Model of Computation with Heterogeneous Time Domains
- Coupling Effects in Process Networks
- Nondeterminism and Stochastic Processes